UTILISING FUZZY INTERPOLATION BEZIER CURVES FOR ALPHABET VERIFICATION

Rozaimi Zakaria[1*], Soon Yun Yee[2]

[1,2]Faculty Science and Natural Resources,
Universiti Malaysia Sabah, 88400 Kota Kinabalu, Sabah, MALAYSIA.

Corresponding author's email: rozaimi@ums.edu.my (Tel: 088-320000 ext: 5628, Fax: 088-320223)

ABSTRACT. *In this paper, alphabet verification is conducted using fuzzy interpolation Bezier curves. Uncertain data can be defined by using the fuzzy number concept. Firstly, Fuzzification in the form of triangular fuzzy numbers is discussed. Then, the defuzzification process is implemented to produce crisp fuzzy data points. An error is obtained by comparing the defuzzified model of alphabet verification with the crisp model. The small error value obtained indicates that the fuzzy interpolation Bezier curve model is acceptable and can be used in modeling alphabet verification.*

KEYWORD. Fuzzy interpolation Bezier curve, alphabet verification, uncertainty data, alpha–cut triangular fuzzy number, defuzzification process

INTRODUCTION

Geometric modeling is a method that is applicable to the construction of a mathematical representation of the geometry under consideration. Fuzzy set theory uses mathematical representations to build the concepts and techniques to solve problems related to uncertain data. Both of these applications have been developed and widely used in various fields such as science, design and engineering (Wahab, 2008; Zakaria, 2010).

However, there are many problems that can arise in the reconstruction of curves due to the uncertainty of the data (Zakaria, *et al*. 2016; Wahab & Zakaria, 2015; Zakaria & Wahab, 2014, 2012). This phenomenon meant that researchers were unable to determine the uncertainty in the data by utilising function curves and surfaces., One way to determine uncertain data is through fuzzy set theory, which was devised by Zadeh in 1965. Fuzzy set theory is different from the theory set of probability. This is because fuzzy set theory is designed to control the uncertain values. and has membership function in the range [0,1].

Furthermore, fuzzy set theory is used to solve the problem of uncertain data. Therefore, to determine the uncertainty in modeling the geometry data, uncertainty data can be resolved by using the concept of fuzzy numbers. The concept of fuzzy numbers is used to define the uncertainty of the data points will be fuzzy data point that will solve the problem in a curve and surface modeling using uncertainty data points (Zakaria, *et al*. 2016; Wahab & Zakaria, 2015; Zakaria & Wahab, 2014; Karim, *et al*. 2013).

Fuzzy set theory is used to define uncertainly data so that the uncertainly data will become fuzzy data points that allow the uncertainly data to be analyzed and visualized (Zakaria & Wahab 2014; Wahab & Zakaria, 2015; Zakaria & Wahab, 2012). Furthermore, a new approach is suggested using alpha cut triangular fuzzy numbers in alphabet verification.

In their research, Roslan & Yahya (2015) implemented a new reconstruction method for alphabets using a cubic Bezier with differential evolution. In this study, the method of differential evolution (EP) is used to find the optimal control point using a cubic Bezier curve. The value of the control points obtained is used in the equation cubic Bézier curve and the number of errors between the actual images with the new parametric curve is calculated using the total squared error.

However, the method discussed above does not utilise the defuzzification process to obtain a crisp fuzzy curve. The fuzzification and defuzzification processes were not elaborated on in detail. Therefore, this paper will examine and explain the fuzzification and defuzzification processes more fully.

This paper proposes alphabet verification based on fuzzy interpolation Bezier curve and alpha-cut triangular fuzzy numbers. The original or crisp alphabet will become the main reference in the modelling alphabet verification, while the fuzzy alphabet verification will undergo fuzzification process to reduce the interval between left alphabet verification and right alphabet verification. After that, the defuzzification process will be carried out to produce crisp fuzzy data points.

MATERIALS AND METHODS

Fuzzy set theory for defining uncertainly data was introduced by Zadeh in 1965 (Zadeh, 1965). He introduced $\vec{A}$ the fuzzy set theory in a work called "fuzzy sets". In order to model the uncertainty of these data points, fuzzy set theory is used to determine the uncertainty of the data points by using the definition of fuzzy numbers when faced with the uncertainty of data in real numbers (Zakaria & Wahab, 2014).

Definition 1: Assume that X is a universal set and A is a subset of X that can be represented as $A \subset X$. Set A is a fuzzy set denoted by $\vec{A}$ if for every $x \in X$ there exists $\mu_A : X \to [0,1]$ which is a membership function that characterizes the membership grade for every element of A in X defined by

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x \in A \quad \text{(full membership)} \\ 0 < c < 1 & \text{if} \quad x \overset{\leftrightarrow}{\in} A \quad \text{(non-full membership)} \\ 0 & \text{if} \quad x \notin A \quad \text{(non-membership)} \end{cases} \tag{1}$$

and c are the membership values. Hence, fuzzy set can be represented in the form $\vec{A} = \{(x, \mu_A(x)) : x \in X\}$ where $\vec{A}$ in X has the membership grade in the range [0,1] (Zakaria & Wahab, 2014; Zakaria *et al*., 2014).

Definition 2: Assume that set $A = \{(X,Y), x \in X, y \in Y \mid x, y : \text{is fuzzy data}\}$ and $\tilde{A} = \{P_i \mid P \text{ is data point}\}$ is a set of fuzzy data points where $A \in A \subset X \times Y \subseteq R$ and R is the universal set where its membership function is $\mu_P(A_i) : A \to [0,1]$ which can be defined as $\mu_P(A_i) = 1$ with $A = \{(A_i, \mu_A(A_i)) \mid A_i \in R\}$. Hence,

$$\mu_P(A_i) = \begin{Bmatrix} 0 & \text{if} & A_i \notin R \\ c \in (0,1) & \text{if} & A_i \overset{\leftrightarrow}{\in} R \\ 1 & \text{if} & A_i \in R \end{Bmatrix} \tag{2}$$

with $\mu_A(A_i) = \left\langle \mu_P(\overleftarrow{A_i}), \mu_P(A_i), \mu_P(\overrightarrow{A_i}) \right\rangle$ where $\mu_A(\overrightarrow{A_i})$ and $\mu_A(\overleftarrow{A_i})$ are left membership grade values and right membership grade values respectively, which can be summarized as

$$A = \{A_i = (x_i, y_i) \mid i = 0,1,...,n\} \tag{3}$$

for every *i*, $A_i = \left\langle \overleftarrow{A_i}, A_i, \overrightarrow{A_i} \right\rangle$ with $\overleftarrow{A_i}$, A_i and $\overrightarrow{A_i}$ are the left fuzzy control point, crisp control point and right fuzzy control point respectively (Zakaria *et al.*, 2016; Wahab & Zakaria, 2015; Zakaria & Wahab, 2014; Zakaria *et al*., 2014).

Triangular fuzzy number is used to define the uncertain data in the form of interval. Below, the method of defining triangular fuzzy numbers is explained.

Definition 3: Triangular fuzzy numbers can be defined using the three points whereby $\overrightarrow{A} = (a,d,c)$ and $\overleftarrow{A_\alpha}$ is the alpha-cut operation of triangular fuzzy numbers. Then, the crisp interval obtained after alpha-cut operation is $\overleftarrow{A_\alpha} = [a^\alpha, c^\alpha] = [(d-a)\alpha + a, -(c-d)\alpha + c]$ with $\alpha \in (0,1]$. Below is a membership function of triangular fuzzy numbers.

$$\mu_{\overleftarrow{A_\alpha}}(x) = \begin{cases} 0 & \text{if} \quad x < a \\ \dfrac{x-a}{d-a} & \text{if} \quad a \le x \le d \\ \dfrac{c-x}{c-d} & \text{if} \quad d \le x \le c \\ 0 & \text{if} \quad x > c \end{cases} \tag{4}$$

a and *c* are the left fuzzy number and right fuzzy number respectively, while d is a crisp point in the interval (Zakaria & Wahab, 2012).

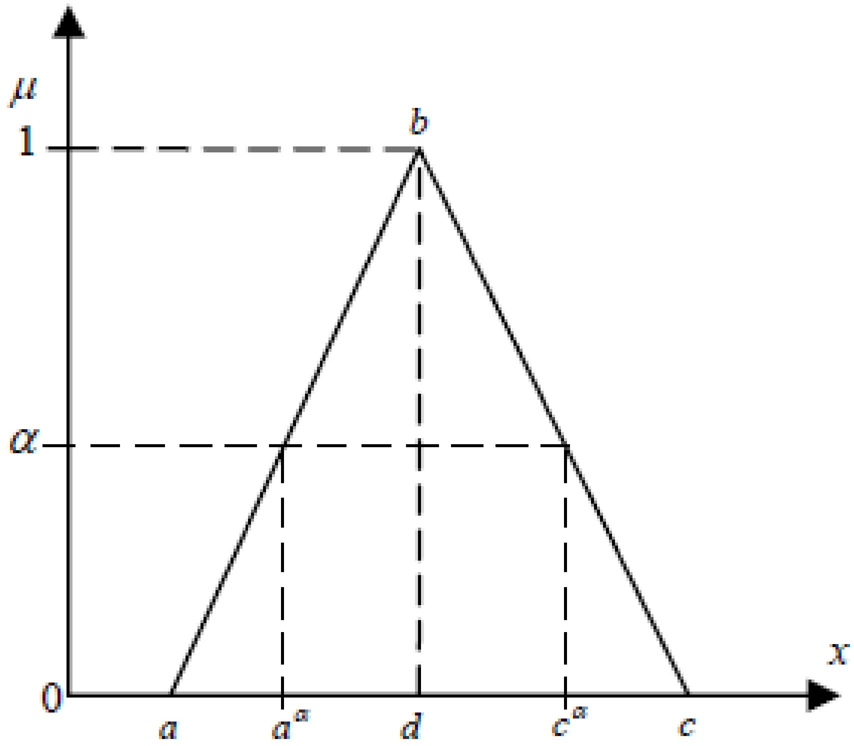

Figure 1: Triangular fuzzy number, $\overrightarrow{A} = (a,d,c)$.

Based on Figure 1, α symbol represents as α values of triangular fuzzy number, α-cut. The higher the α values, the smaller the distance between a^α and c^α. Furthermore, the value *b* is the crisp fuzzy number, which has a full membership function that is equal to 1.

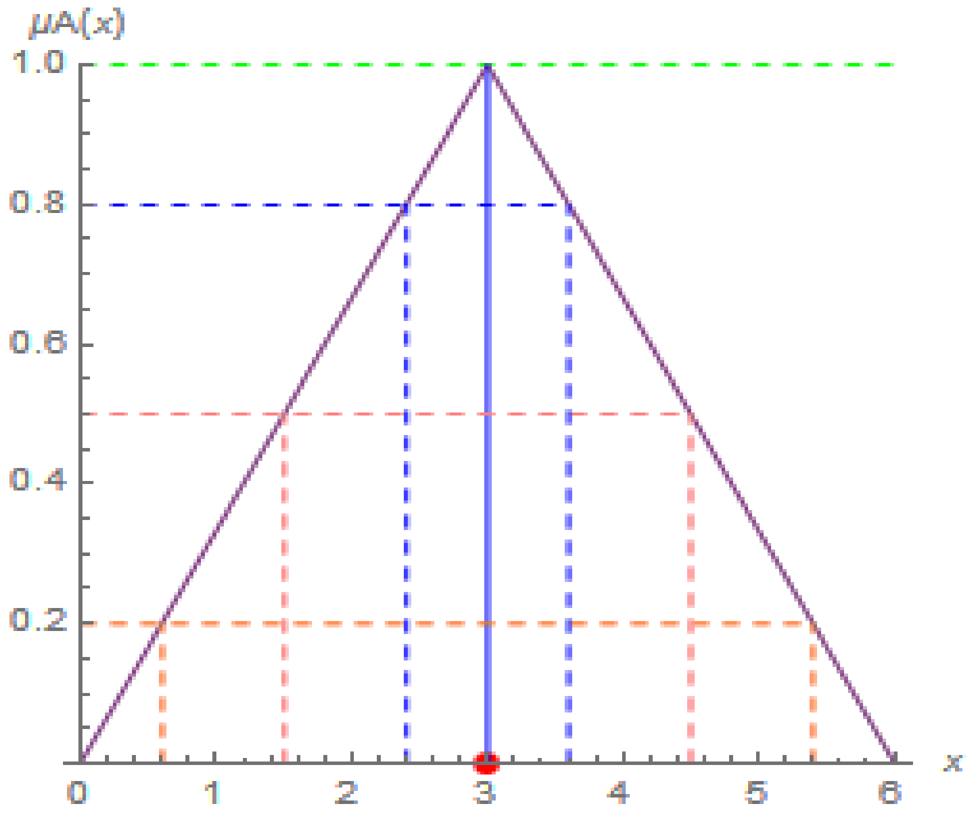

Figure 2: Alpha-cut triangular fuzzy number with different alpha values.

Figure 2 shows that different alpha values will produce different confidence fuzzy intervals. The higher the value of alpha-cut, the smaller the confidence fuzzy interval.

Based on Definition 3, we can use the alpha-cut operation of triangular fuzzy number to do the fuzzification process. The definition of fuzzification process of fuzzy control points can be summarized as follows:

Definition 4: Assume that α is alpha-cut operation for every fuzzy data point, P_i with $P_i \in P$. Hence, alpha-cut operation, also known as fuzzification process can be defined by the following equation:

$$P_{i_\alpha} = \left\langle P_{i\,\alpha}^{\leftarrow}, P_i, P_{i\,\alpha}^{\rightarrow} \right\rangle \tag{5}$$

Where $P_{i\,\alpha}^{\leftarrow}$ and $P_{i\,\alpha}^{\rightarrow}$ are the left fuzzy data point value and right fuzzy data point value respectively after alpha-cut operation while P_i is the crisp data point for every $\alpha \in (0,1]$ and $i = 0,1,...,n$ (Zakaria *et al.*, 2014).

Definition 5: Assume that $P_{i_{\alpha_k}}$ is the fuzzy data point after alpha-cut operation and P_{α_k} is the defuzzify data point for $\tilde{P}_{i_{\alpha_k}}$ as shown below (Zakaria *et al.*, 2014).

$$P_{\alpha_k} = \left\{ P_{i_{\alpha_k}} \right\} \tag{6}$$

Where $P_{i_{\alpha_k}} = \frac{1}{3}\sum_{i=0} \left\langle \tilde{P}_{i\,\alpha_k}^{\leftarrow}, P_i, \tilde{P}_{i\,\alpha_k}^{\rightarrow} \right\rangle$.

The defuzzification process is shown in Figure 3.

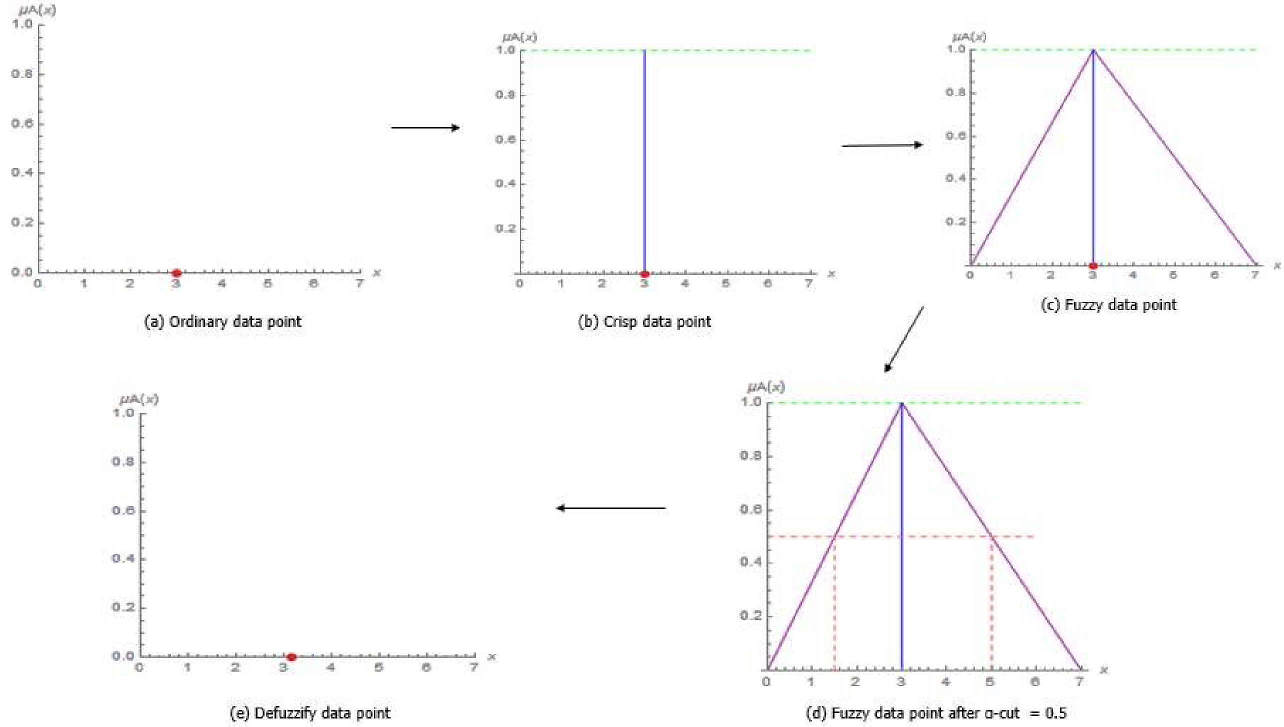

Figure 3: Defuzzification process of fuzzy data points

RESULT AND DISCUSSION

The alphabet verification must first be digitized via scanner and then analyzed to obtain the data point. Figure 4 shows ten samples of the same alphabet with small but observable differences that are used in this study. Then, the first alphabet becomes the crisp alphabet. This crisp alphabet is the main reference for the other alphabets in order to achieve the objectives of this paper. Fig. 5 is the alphabet modelled using fuzzy interpolation Bezier curve. Furthermore, Fig. 6 shows the alphabet is segmented before fuzzy interpolation Bezier curve is applied.

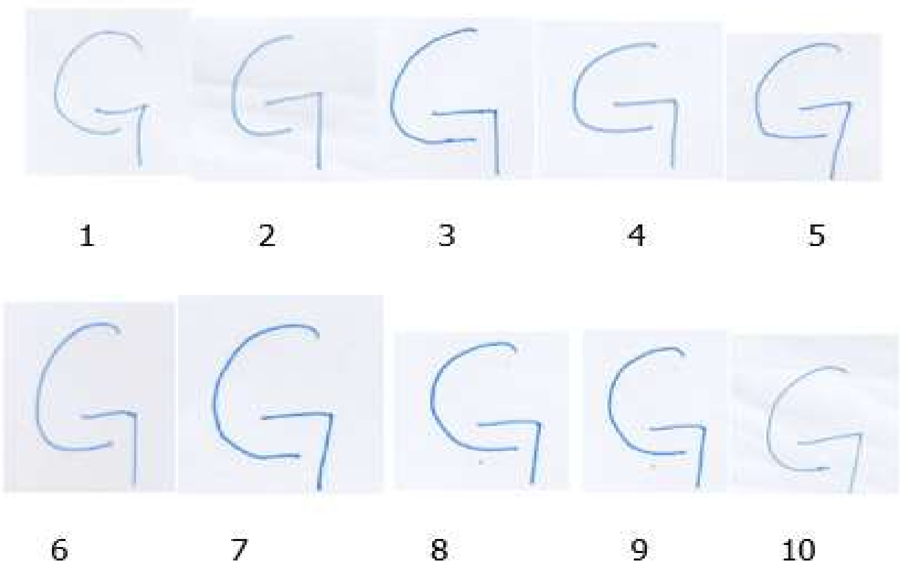

Figure 4: Ten samples of alphabet verification.

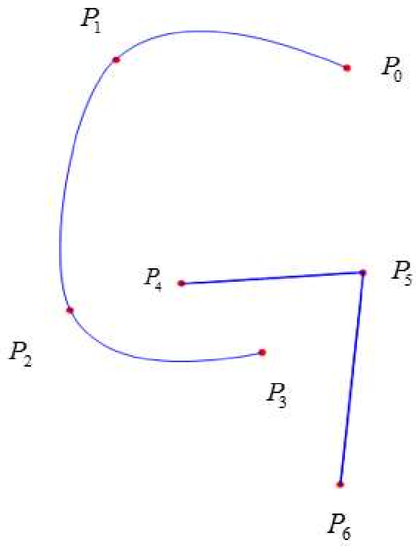

Figure 5: Crisp model of alphabet verification.

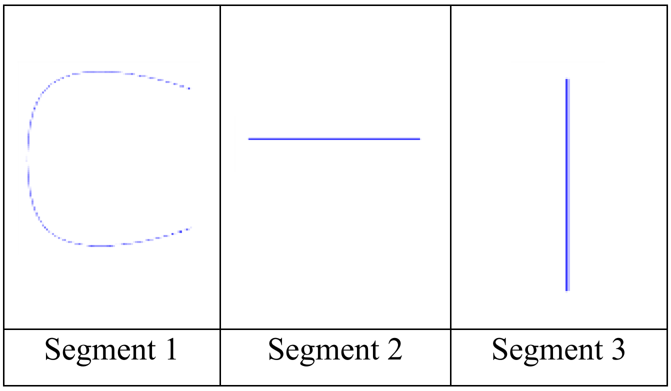

Figure 6: Segments of alphabet verification.

Figure 7 shows ten samples of alphabet modelled in the same axis. The blue alphabet represents the crisp alphabet verification. The ten samples alphabet modelled in same axis to get the upper bound and lower bound of fuzzy alphabet. The lower and upper bound of fuzzy alphabet is created based on the crisp alphabet modelled in Figure 5.

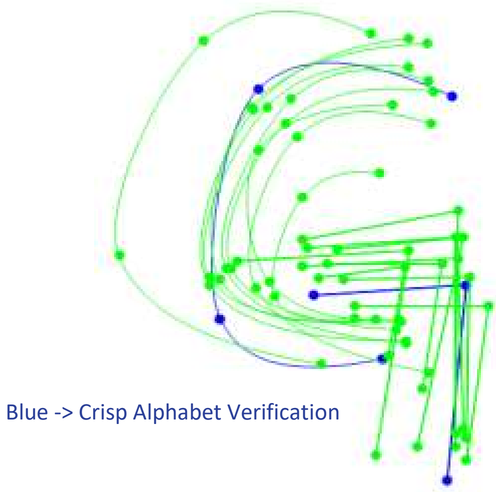

Figure 7: Ten samples of alphabet modelled in the same axis.

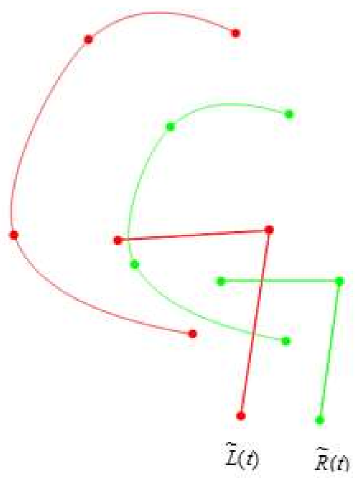

Figure 8: Lower and upper bound of fuzzy alphabet taken by using alpha-cut operation of triangular fuzzy number.

Figure 8 shows the upper bound (right) and lower bound (left) fuzzy interpolation Bezier Curve for modelling alphabet "G". The purpose for model the right and left of the fuzzy interpolation Bezier curve of alphabet verification is to reduce the number of the alphabet and can produce one of the best alphabets that can be used in the verification of the alphabet "G".

Figure 4 shows that the ten samples of the alphabet are numbered to 1 until 10. The first alphabet will be crisp alphabet. Indirectly, the crisp alphabet will also be used as reference for another nine samples of the alphabet in order to achieve the objectives of the study. This study can also be conducted by using a different alphabet.

Figure 9-11 show fuzzification of alphabet verification by using fuzzy interpolation Bezier curve with different alpha value such as $\alpha = 0.2$, $\alpha = 0.5$ and $\alpha = 0.8$ respectively.

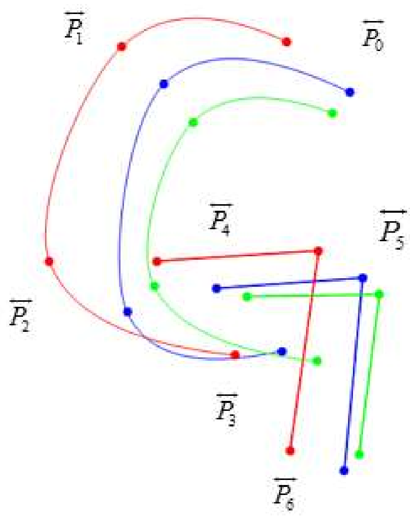

Figure 9: Fuzzify alphabet verification model using fuzzy interpolation Bezier curve with $\alpha = 0.2$.

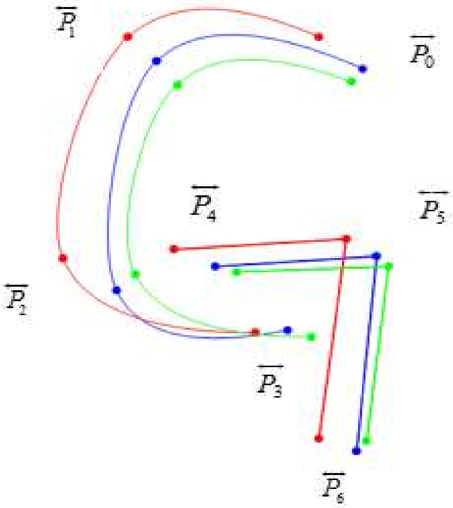

Figure 10: Fuzzify alphabet verification model using fuzzy interpolation Bezier curve with $\alpha = 0.5$.

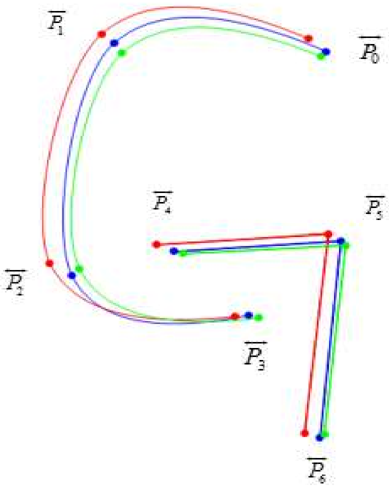

Figure 11: Fuzzify alphabet verification model using fuzzy interpolation Bezier curve with $\alpha = 0.8$.

After process fuzzification, the aphabet model will undergoes process defuzification. Fig. 12 show the defuzzify alphabet verification model with alpha-cut value is $\alpha = 0.5$. The defuzzification process can be refer based on Definition 5.

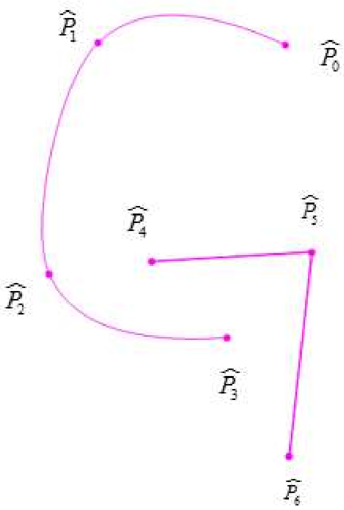

Figure 12: Defuzzify alphabet verification model using fuzzy interpolation Bezier curve with $\alpha = 0.5$

DISCUSSION AND CONCLUSION

To investigate the effectiveness of the output of alphabet verification, the error between the defuzzified data points and crisp data points of alphabet verification need to be known and can be defined as follows (Zakaria *et al.*, 2014).

$$\frac{\sum_{i=1,\dots,n}^{n} {}^{G}P_{i_e}}{n} \tag{7}$$

where ${}^{G}P_{i_e} = \frac{{}^{G}P_i - {}^{G}P_i}{{}^{G}P_i}, \quad i = 0,1,2,\dots,n = 7$.

The alphabet verification can be constructed based on the fuzzy set theory, fuzzy number and fuzzy interpolation Bezier curve. The alphabet verification model using fuzzy interpolation Bezier curve can be executed with an error of less than 0.1. Error is acceptable into this study suggest that this study is valid and can be used by another researcher in the study of fuzzy geometric modeling.

Table 1 shows the the error between the defuzzified alphabet model and crisp alphabet model. The value of errors is 0.00123554 in x-axis and 0.00630467 in y-axis. Hence, this error is acceptable because it is in the range of less than 0.1.

Table 1: Error of alphabet verification.

Alpha-cut value	Axis	
	X	Y
$\alpha = 0.5$	0.00123554	0.00630467

Alphabet verification using an alpha cut of triangular fuzzy number has been constructed. Fuzzy interpolation Bezier curve model was used in order to design the alphabet verification model. Besides, different definitions and equations used in this study to ensure the success of fuzzy interpolation Bezier curve to make an alphabet verification model. The fuzzification process is applied to obtain the fuzzy interval of fuzzy data points, then the defuzzification process is followed to find a crisp interpolation Bezier curve.

ACKNOWLEDGEMENT

The authors would like to acknowledge Universiti Malaysia Sabah and Ministry of Higher Education (MOHE), Malaysia, for providing the facilities to conduct this research. They further acknowledge anonymous referees for their constructive comments which improved the readability of this paper.

REFERENCES

Wahab, A.F. 2008. Pemodelan Geometri Menggunakan Teori Set Kabur, Universiti Sains Malaysia.

Karim, N. A. A., Wahab, A. F., Gobithaasan, R. U., & Zakaria, R. 2013. Model of Fuzzy B-Spline Interpolation For Fuzzy Data. *Far East Journal of Mathematical Sciences (FJMS)* 72 (2):269-280.

Roslan, N. & Yahya, Z. R. 2015. Pembinaan Semula Fon dengan Bezier Kubik Menggunakan Evolusi Pembezaan. *Sains Malaysiana* 44 (8):1203-1208.

Wahab, A. F., and Zakaria, R. 2015. Fuzzy Tuning B-spline Curve. *AIP Proceedings*:1-7.

Zadeh, L. 1965. Fuzzy Sets. *Information and Control* 8:338-353.

Zakaria, R. 2010. Pemodelan Interpolasi Splin Bezier Kabur, Jabatan Matematik, Universiti Malaysia Terengganu.

Zakaria, R. and Wahab, A. F. 2012. Fuzzy B-spline Modeling of Uncertainty Data. *Applied Mathematical Sciences* 6 (140):6971-6991.

Zakaria, R. 2014. Pemodelan Titik Data Kabur Teritlak. *Sains Malaysiana* 43 (5):799-805.

Zakaria, R., Wahab, A. F., & Gobithaasan, R. U. 2014. Fuzzy B-Spline Surface Modeling. *Journal of Applied Mathematics* 2014:8 pages.

Zakaria, R., Wahab, A. F., & Gobithaasan, R. U. 2016. The Series of Fuzzified Fuzzy Bezier Curve. *Jurnal Teknologi* 78 (2-2).