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Research Article

Population growth model for *Blattella bisignata* cockroach

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ABSTRACT. This paper aims to present the logistic population growth model for *Blattella bisignata* cockroach based on experimental data, conducted in the Botanical Laboratory of the School of Science and Technology, Universiti Malaysia Sabah. Parameters involved in this experiment include the types and amounts of food given, temperature and water. The step sizes for future population estimations are calculated using logistic model techniques with varying values of the reproductive parameter, r. Maple 12 is used to simulate the results in the range of r between 0.1-2.0. The population dynamics of *Blattella bisignata*, show chaotic dynamics as r approaches 2.0. Therefore, understanding and identifying factors which underlie the stochastic behaviour of *Blattella bisignata* would give further insight into the importance of the chaotic dynamics and ecological balance of the insect population.

Keywords: Population dynamics, logistic growth model, chaotic dynamics, ecology.

INTRODUCTION

Populations of organisms tend to increase as far as their environment will allow. As a result, most populations are in a state of dynamic equilibrium. Their numbers increase in a delicate balance that is influenced by limiting factors. Population dynamics of a specific species are determined by these underlying factors. These factors, in general, are the level of nutritional components, crowding and competition, and waste concentration increase (Al-Khaled, 2005).

By definition, population dynamics is the study of the numbers of populations and the variations of these numbers in time and space. It is the branch of life sciences that studies short- and long-term changes in the size and age composition of populations, and the biological and environmental processes which influence these changes. Also, it deals with matters concerning populations' birth and death rates, immigration and emigration, and topics such as ageing populations or population decline. Most populations are in a dynamic state of equilibrium as populations of organisms tend to increase as far as their environment will allow, and are influenced by limiting factors as described above.

Population growth modelling in insects is an application derived from population dynamics. It is widely used to study the relationship between a certain population and specific mathematical equations, in this case the study of *Blattella bisignata* cockroaches. This cockroach is among the most widespread household pests in the world, and is now almost impossible to sustain itself outside human constructions (Wu *et al.*, 2007).

In this paper, we will examine the range of the reproduction parameters which affect the population growth model. We also hope to identify points of stability and chaos of the population dynamics.

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Brief description of *Blattella bisignata*

In Malaysia, several cockroach surveys have been reported (Oothuman *et al.,* 1984; Yap *et al*., 1991; Zahedi & Jeffery, 1996; Lee & Lee, 2000). The most abundant genus is *Periplaneta*, followed by *Blatella.* The German cockroach, *Blattella germanica* is one of the most widespread household pests in the world, while *B. bisignata* is limited to the Southeast Asian region, and mostly lives in open spaces (Wu *et al.,* 2007). The study of *B. bisignata* has increased since the findings of Wu & Shiao (1994) in Taiwan on its morphological discrimination from *B. germanica.*

The life cycle of *Blattella bisignata* is similar to other cockroach species, and consists of three developmental stages: eggs, nymphs, and adults (Roth, 1991). Eggs develop in an ootheca, which protrudes from the posterior end of a female and are carried by the female before the nymphs hatch (Figure 1). The number of molts that is required to reach adulthood varies, and the most frequently reported number of molts is six.

Adult males and females need a couple of days to become receptive. Males then stay virtually continuously receptive until they die. On the other hand, females go through a number of reproductive cycles. They have to find a reproductively active male, mate during a short mating window, spend some time carrying ootheca, and finally recover to become receptive again. Even females that do not succeed in mating within the mating window will become pseudo-pregnant and need to recover after that. Once mated, the females will store the sperm, and need not mate again in order to develop another fertile ootheca. However, not all mating will result in a fertile ootheca and one mating is often not sufficient to supply enough sperm to avoid the need of new mating (and hence another mate search) later in their life (Wu *et al.,* 2007).

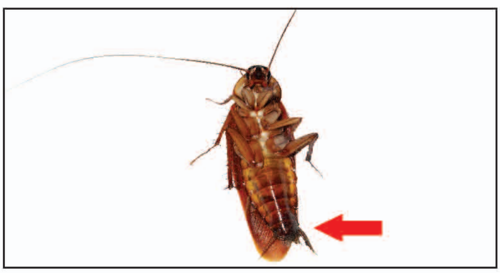


Figure 1. Female *Blattella bisignata* with ootheca.

METHODOLOGY

Blatella bisignata was collected from the vicinity of the city of Kota Kinabalu, Sabah, and population growth experiments were conducted in the Botanical Laboratory of the School of Science and Technology, Universiti Malaysia Sabah. They were maintained for many generations in environmental chambers. Detailed information about rearing conditions has been given previously by Lee & Wu (1994).

In Wu (2007), the female mating rate is a function of the number or density of receptive males,

$$\phi_i = k_i(1-\exp(-S_d M_a)) \quad (1)$$

,where k_i is the probability of a receptive female to mate on the *i*th day of the female window provided it still needs mating activities, and S_d is the effective area searched by the female per day. This expression precisely holds if the males are randomly distributed with density M_a , and the females will search randomly until a male 'falls' within the female perception range, in which case the female goes directly to the male. We can therefore write:

$$S_d = \int_{1day} S(t)d(t) \quad (2)$$

,where *S(t)* is the effective area searched within a small time interval *(t, t+dt)*. As both sexes have to be simultaneously active to initiate copulation, we can then write,

$$S(t)=a(L_M(t)+L_F(t)) \quad (3)$$

,with *S(t)*=0 and at least one of the sexes is not active. Here, L_M *(t)* and L_F *(t)* quantify locomotion activity of the males and females, respectively, scaled to the maximum of one-half (to have their sum scaled to the maximum of one). The 'movement efficiency' constant '*a*' is introduced into equation (3) so as to account for this scaling. Also, it comprises other features which affect the female mating rate, for example, the degree of species gregariousness, individual perception range, movement rate, fraction of receptive males and the number of females active at any time, etc. All these features are currently assumed time invariant, although the model is open to any changes in this respect.

In this paper, the logistic model of the population dynamics is being applied. As in Haeussler *et al.* (2005), the rate of change of a certain population can be represented by the following equation:

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right), \quad (4)$$

,where $N(t)$ is the population concentration, K is the environmental carrying capacity, and r is the reproduction parameter.

The reproduction parameter, r, in equation (1) is said to be not a constant at all times, but rather will decrease with changes in time, population size or even population concentration. Expanding equation (1) will result in the following equation as:

$$\frac{dN(t)}{dt} = rN(t) - \frac{r}{K}N^2(t) \quad (5)$$

This is in the form of $P'(t)=[a-b.P(t)]P(t)$, and for all a, b>0 (Buchana & Carlos, 1992). The birth rate dN/dt is still a constant, giving a value of rN. However, in population modelling, every population will also experience birth-death rates, thus rN^2/k will give a negative effect especially as the population increases. The initial value of any respective population is taken as $N(0) = N_0$.

The input variables in determining the population dynamics will include the types and daily amounts of food and water given as well as the temperature range subjected to the samples. These reproduction experiments are conducted over two life cycles of the cockroach, that is, approximately 200 days. The birth-death rates can then be determined from the experimental information obtained.

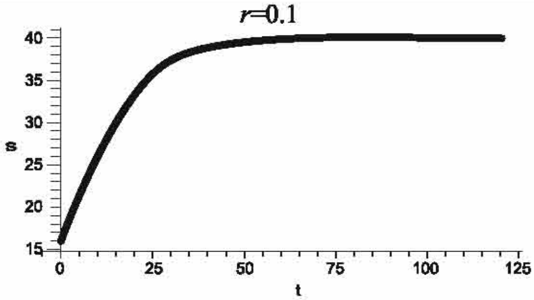
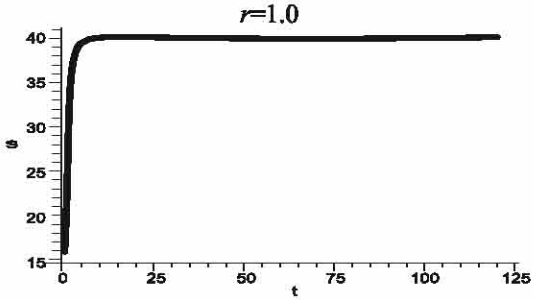
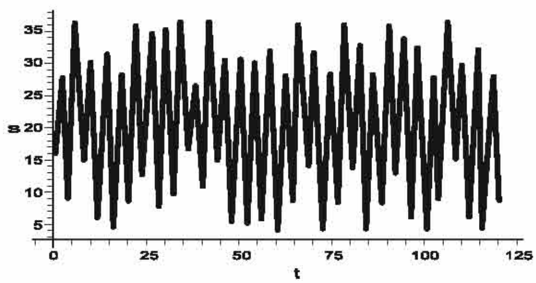
Using MAPLE 12, the logistic models are then simulated using different values of r ranging from 0.1 to 3.5. Simulations of the trends in population dynamics can give us some indication of future population estimations.

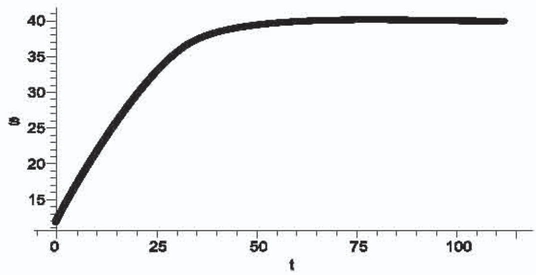
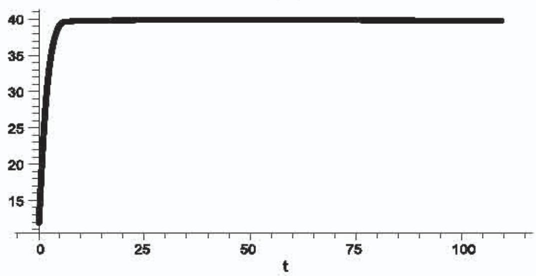
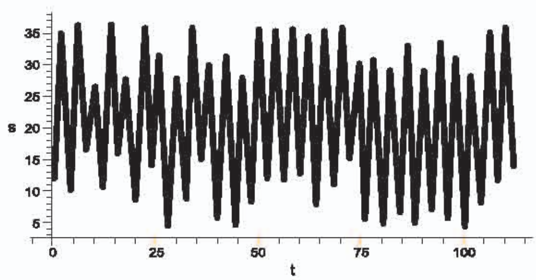
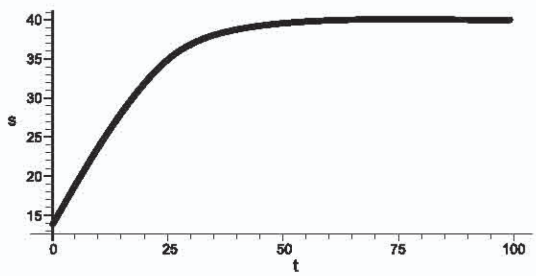
RESULTS AND ANALYSES

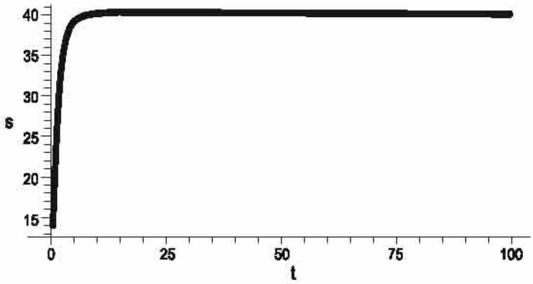
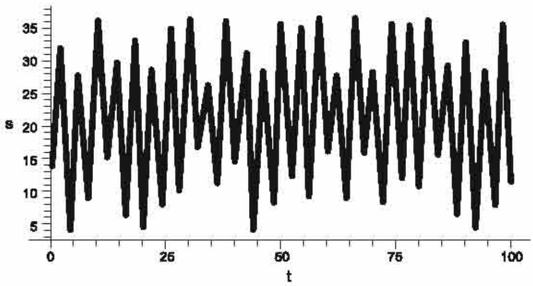
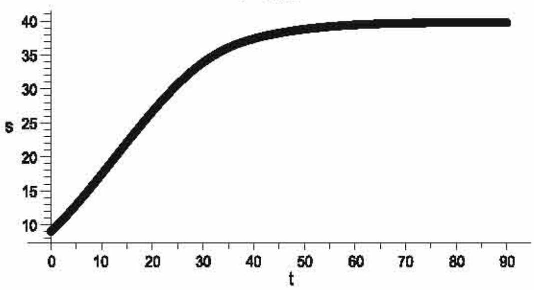
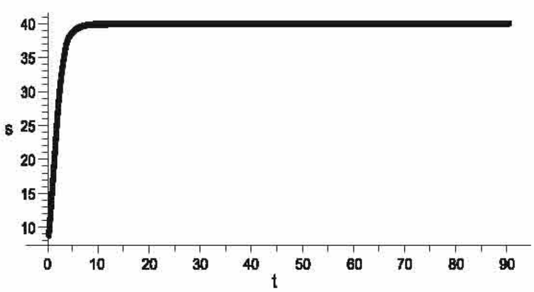
The logistic population growth models were implemented using data obtained from experiments to make predictions of the population growth of *Blattella bisignata*. The reproduction parameter, r, is represented by the step size with respect to time so as to approximate the cockroach population. Daily outputs of trends of the models as the range of r is varied are observed, before the dynamics of the models then turn to become chaotic. The trends then start to show some chaotic dynamics for ten samples (S1-S10) as r approaches 2.0. However, samples S11 and S12 respectively, have started to show some stability in their logistic dynamical trends, where the stability points have reached beyond 2.0 (r >2.0). In other words, the population dynamics of these particular samples have managed to optimize their inputs, without subjecting to cannibalism.

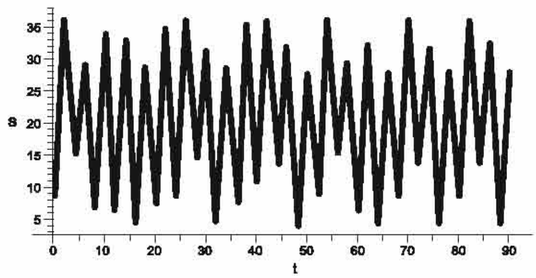
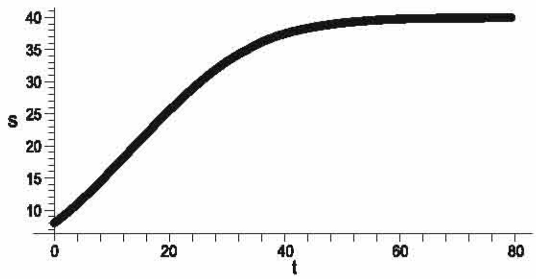
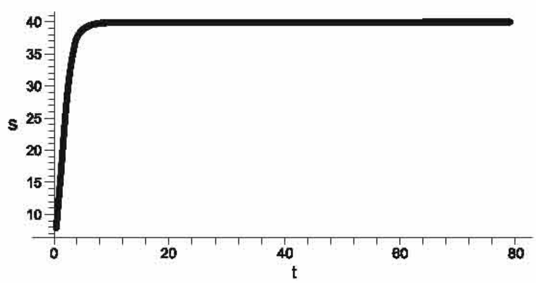
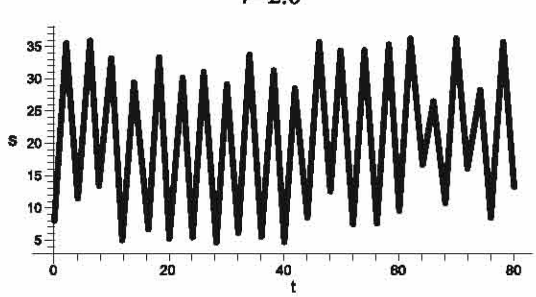
The outputs and analyses of the logistic model trends is demonstrated graphically (Table 1) for 12 samples with selected reproduction parameters, r, whereby r=0.1, r=1.0, and r=2.0 are respectively observed. The graphs exhibit outputs of the logistic population growth models of *Blattella bisignata* for the 12 samples. It is observed that when the reproductive activities are active with null chaos and no perturbations being exhibited, the dynamics of the models are therefore controlled and growth optimality is thus achieved. This also correlates with the linearly positive trends of the models. The optimal conditions for this phenomenon to occur are in the temperature range of 18°C - 24°C, and a moisture content of 0.275ml per day. Since *Blattella bisignata* will scavenge on practically any type of food, the amount of food is more a determining factor for its survival. This is exhibited by the cannibalistic behaviour of *Blattella bisignata* when the birth rate escalates rapidly, and the reproductive environments are no longer conducive. These phenomena have been shown by the representation of most of the samples (S1-S10) above. The reproductive parameter, *r*, also indicates the insect's living environments, as such, it could be a measure for ecologists in making estimations and predictions of the insect's population.

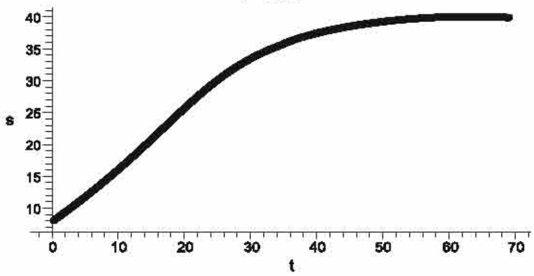
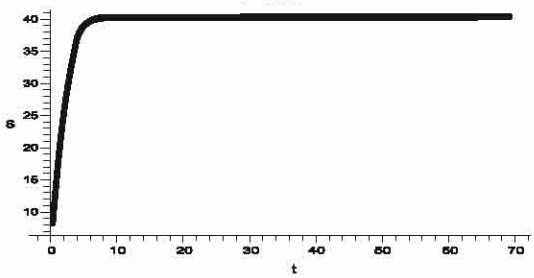
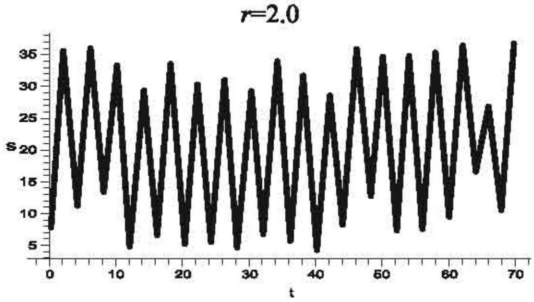
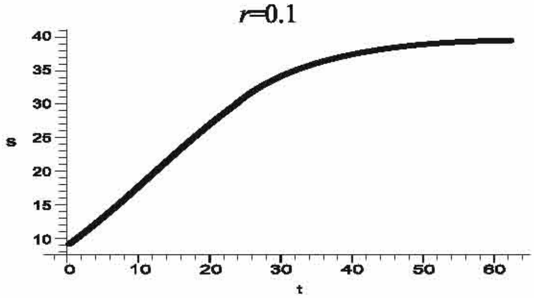
Table1. Graphical outputs and analyses of the population logistic models.

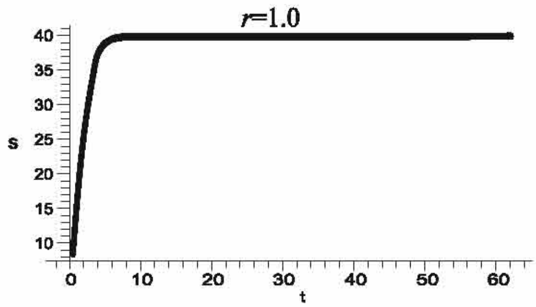
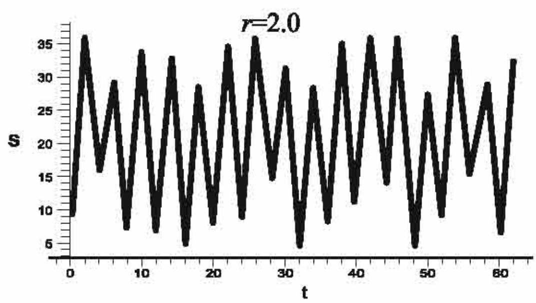
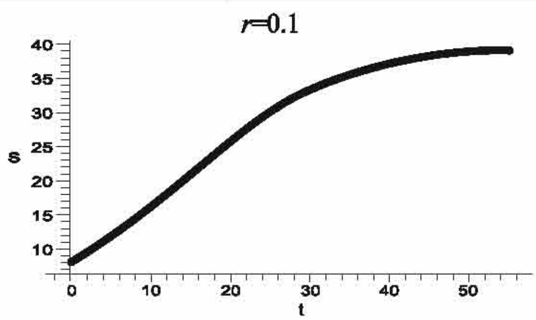
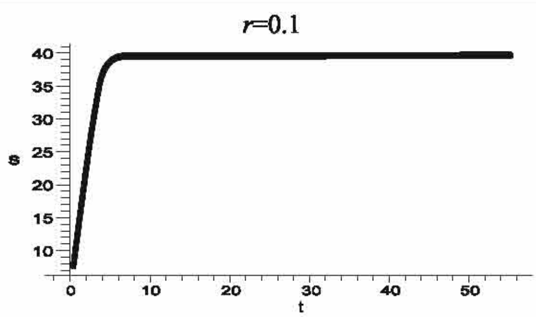
<table>
<tr><th>Sample Description</th><th>S1; N=16; No. of Days =120</th><th>Model Analyses</th></tr>
<tr><td colspan="2">r=0.1

</td><td>Starting with r=0 an increasing but gentle slope indicates the positive relationships of all the input variables in the model. As r reaches 0.1, it appears to reach a constant value of about 0.79 mating per day.</td></tr>
<tr><td colspan="2">r=1.0

</td><td>There is a steep slope of value approximately 8.0, as r approaches 1.0 and towards 2.0. This effect is due to the increase in the number of mating and reproduction, and the number of males and females in the population.</td></tr>
<tr><td colspan="2">r=2.0

</td><td>As r ~2.0, the graph tends to oscillate about the axis of S=16.25 mating with an amplitude of 32.5. The model has started to exhibit chaotic dynamics where its point of stability is in the range of between 1.0 and 2.0. The chaotic cycle is estimated at 26 per 100.</td></tr>
</table>

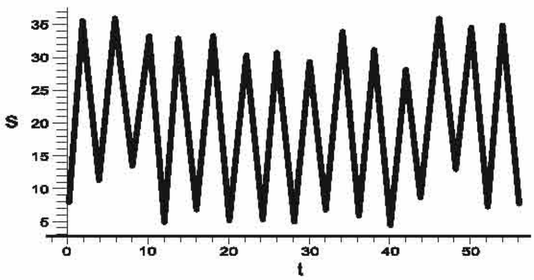
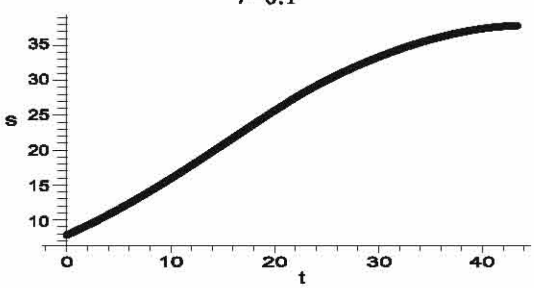
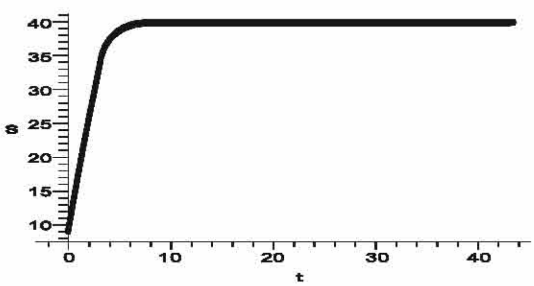
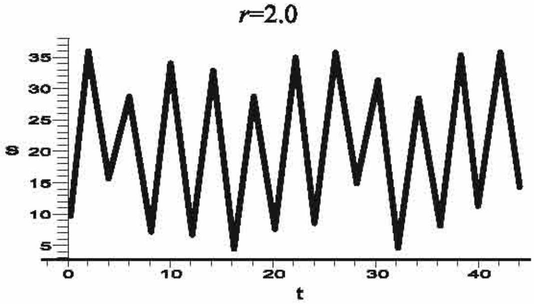
Sample Description S2; No. of Days =111; N=11	**Model Analyses**
r=0.1 	Similarly, with S1, the trend has increased gently, and tends to reach a constant value of 0.78 mating per day.
r=1.0 	A steep gradient with similar effects as to S1 is observed.
r=2.0 	Oscillations are about the axis of S=16.5 mating with an amplitude of 33. Chaos is exhibited as r approaches 2.0 of an estimated cycle of 26 in 100.
Sample Description S3; No. of Days =99; N=14	**Model Analyses**
r=0.1 	A similar trend as S1 and S2 are exhibited.

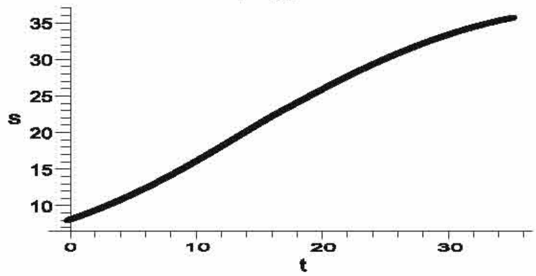
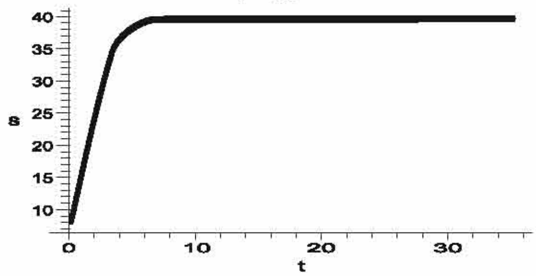
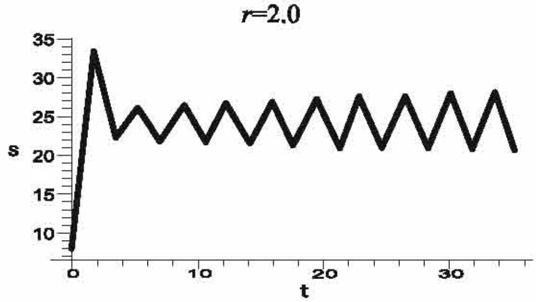
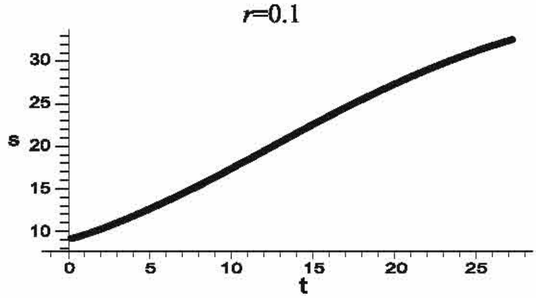
r=1.0 		A steep gradient which is similar as S1 and S2.
r=2.0 		Oscillations about the axis S=16.5 mating. Chaos is exhibited with a cycle of 25 in 100.
Sample Description	S4; No. of Days =90; N=9	**Model Analyses**
r=0.1 		A similar trend is observed, but a lesser value of 0.65 mating per day.
r=1.0 		A steep gradient of about 5.65 is observed.

r=2.0 	Oscillations about the axis S=16.25 mating. The amplitude is 32.5.
Sample Description — S5; No. of Days =79; N=8	**Model Analyses**
r=0.1 	A similar trend is observed at a slightly higher frequency of 0.667 mating per day.
r=1.0 	A gradient of approximately 9.75 is observed.
r=2.0 	Oscillations of amplitude 32.6 at S=16.3 mating. Cycle has decreased to 24.375 per 100.

Sample Description	S6; No. of Days =69; N=8	Model Analyses
r=0.1 		A similar trend is observed, but an increased frequency of 0.727 mating per day.
r=1.0 		A gradient of approximately 5.83 is observed.
r=2.0 		Oscillations cycle of 25 per 100 and amplitude of 31.5 at axis S=15.75 mating.
Sample Description	**S7; No. of Days =62; N=9**	**Model Analyses**
r=0.1 		A similar trend is observed and a frequency of 0.667 mating per day.

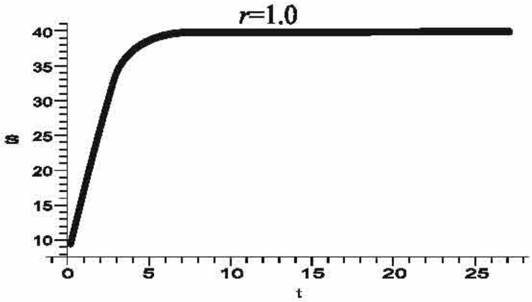
		A gradient of approximately 8.0 is observed
		Chaos and perturbations are still being exhibited. The perturbation cycle is of 25 per 100 with an amplitude of 32.5 at S= 16.25 mating.
Sample Description	S8; No. of Days =55; N=8	**Model Analyses**
		There is an increased frequency of 0.8 mating per day.
		A gradient in the range of 6.7 – 8.0 is observed

		Amplitude and perturbation have reduced to 31.5 and 24.75 cycles respectively.
Sample Description	S9; No. of Days =43; N=9	**Model Analyses**
r=0.1 		A slope of 0.86 mating per day is observed.
r=1.0 		A gradient in the range of 6.7-8.0 is observed.
r=2.0 		Perturbations with less oscillations though. The amplitude has decreased to 32.1 and a cycle of 23.8 per 100.

Sample Description	S10; No. of Days =35; N=8	**Model Analyses**
r=0.1 		An increased slope of 1.13 mating per day is observed.
r=1.0 		A similar steep gradient is observed as in samples S1-S9.
r=2.0 		The amplitude has decreased compared with the other samples to approximately 9.0 only. Oscillations are 30 per 100 at the axis S=24.5 mating.
Sample Description	S11; No. of Days =27; N=9	**Model Analyses**
r=0.1 		A further increase in slope of 1.2 mating per day is observed.

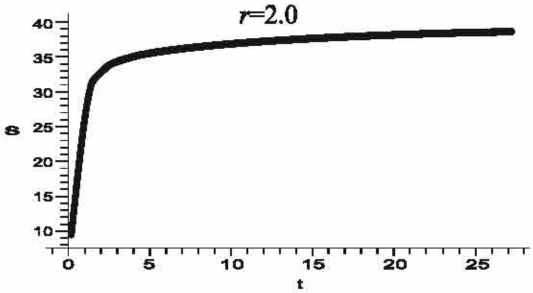
<table>
<tr><td colspan="2">

r=1.0



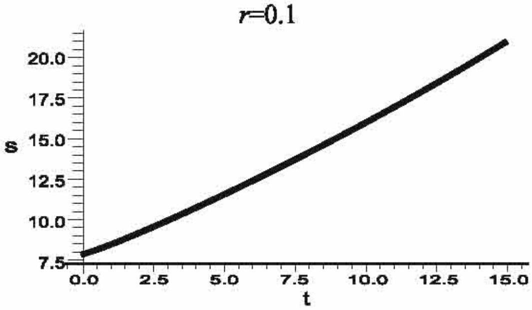
</td><td>A steeper slope still but stability is reached after about 4-5 days.</td></tr>
<tr><td colspan="2">

r=2.0



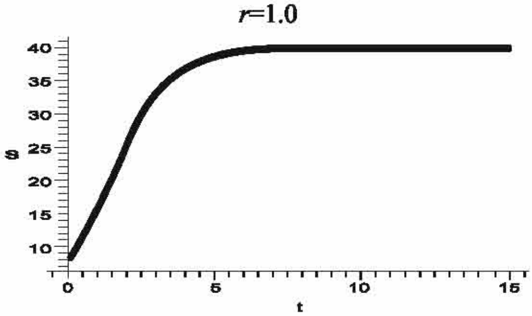
</td><td>A steep slope with stability reached without perturbations and chaotic oscillations exhibited.</td></tr>
<tr><td>Sample Description</td><td>S12; No. of Days =15; N=8</td><td>Model Analyses</td></tr>
<tr><td colspan="2">

r=0.1

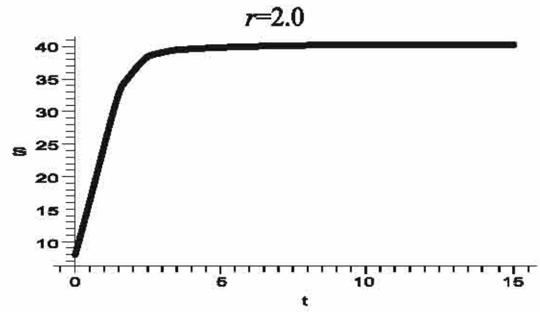


</td><td>A slope of 1.625 is further observed.</td></tr>
<tr><td colspan="2">

r=1.0



</td><td>A gradient of the range 8.0-8.9 before stabilizing to a constant value.</td></tr>
</table>

	A gentle slope than S11. Chaos is no longer exhibited as r reaches 2.0. The dynamics has reached a wider range of r beyond 2.0.

DISCUSSION

The cockroach has a life-cycle of about 100 days. The results have shown that by relating the logistic growth to the reproduction parameter, *r*, the trends of the insect population growth models are found to be constant and quite consistent in their cycles. The range of *r* is from 0-3.5 with a varying interval from 0.5-0.9. It is recommended that a smaller range of intervals be employed where slight changes in trends can be identified.

The numerical computation of the logistic models also suggests that other insect population dynamics can be explored. Modelling and simulating of other insect population dynamics towards a mathematical discipline can ultimately be geared into the understanding of the structure and factors underlying the chaotic behaviour of populations involved. While doing so, their impact on human economies can also be further analysed, such as the production of crickets as fish-feeding stocks for the fisheries industries and grasshoppers as food in Mexico, China, and in some African countries. It is further suggested that more work can be done to obtain the numerical approximations of *Blattella bisignata* population models (Al-Khaled, 2005), while studying its economic and commercial contributions. In other words, large scale production and control of the insect population can be looked on as proactive and productive activities in generating economies for industries, such as in fisheries and aquaculture, pest-control and others.

CONCLUSION

The cockroach (*Blattella bisignata*) exhibits a population growth of a linearly positive trend, where future predictions and estimations can be made based on a range of reproduction parameters. A reproduction parameter in the range of 0-1.99 exhibits controlled population dynamics. Using a wider range of reproduction parameters with different input variables, future studies are suggested with numerical approximations and algorithms that can be developed for a universal application.

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